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THE EQUIVARIANT EULER CHARACTERISTIC OF MODULI SPACES OF CURVES.

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ABSTRACT. We derive a formula for the S_n -equivariant Euler characteristic of the moduli space $\mathcal{M}_{g,n}$ of genus g curves with n marked points.

1. INTRODUCTION

Consider the moduli space $\mathcal{M}_{g,n}$ of algebraic curves of genus g with n marked points. The symmetric group S_n acts naturally on this space. Let V_λ denote the irreducible representation of S_n corresponding to a Young diagram λ , then one can decompose the cohomology of $\mathcal{M}_{g,n}$ into isotypic components:

$$H^i(\mathcal{M}_{g,n}) = \bigoplus_{\lambda} a_{i,\lambda} V_\lambda.$$

The S_n -equivariant Euler characteristic of $\mathcal{M}_{g,n}$ is defined by the formula

$$\chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{i,\lambda} (-1)^i a_{i,\lambda} s_\lambda,$$

where s_λ denotes the Schur polynomial labeled by the diagram λ . We calculate these equivariant Euler characteristics for all $g \geq 2$ and n .

Theorem 1.1. *The generating function for the S_n -equivariant Euler characteristics of $\mathcal{M}_{g,n}$ has the form*

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{\underline{k}} c_{k_1, \dots, k_r} \prod_{j=1}^r (1 + p_j t^j)^{k_j},$$

where p_j are power sums and the coefficients c_{k_1, \dots, k_r} are defined by the equation (6).

Consider the moduli space $\mathcal{M}_g(k_1, \dots, k_r)$ of pairs (C, τ) where C is a genus g curve and τ is an automorphism of C such that for all i the Euler characteristic of the set of points in C having the orbit of length i under the action of τ equals ik_i . The coefficient c_{k_1, \dots, k_r} can be also defined as the orbifold Euler characteristic of $\mathcal{M}_g(k_1, \dots, k_r)$.

This moduli space can be defined for any tuple of integers (k_1, \dots, k_r) of arbitrary size r , but we prove that (for a fixed genus g) it is non-empty only for a finite number of such tuples. In particular, r cannot exceed $4g + 2$.

Corollary 1.2. The generating function $\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n})$ is a rational function in t . Furthermore, for any n ,

$$\chi^{S_n}(\mathcal{M}_{g,n}) \in \mathbb{Z}[p_1, \dots, p_{4g+2}].$$

Theorem 1.1 can be compared with the computations of [4], [5], [8] and [10] in genus 2 and with the computations of [1], [2], [9], [17] and [18] in genus 3. A similar generating function for the moduli spaces of hyperelliptic curves was previously obtained in [11]. The non-equivariant Euler characteristics of moduli spaces of curves were computed by Bini and Harer in [3].

The paper is organized as follows. In Section 2 we consider a complex quasi-projective variety X with an action of a finite group G . Theorem 2.5 provides a formula for the S_n -equivariant

Euler characteristic of quotients $F(X, n)/G$, where $F(X, n)$ is a configuration space of n labeled distinct points on X . This theorem was previously proved in [10] using the results of Getzler [6, 7] concerning Adams operations over the equivariant motivic rings (see also [12]). The alternative proof presented here uses only the basic properties of Euler characteristic and seems to be more geometric. It also makes the proof of the main result self-contained.

In Section 3 we apply this theorem to the universal family over \mathcal{M}_g , the moduli space of genus g curves. This allows us to prove in Theorem 3.3 that the coefficient c_{k_1, \dots, k_r} is equal to the orbifold Euler characteristic of $\mathcal{M}_g(k_1, \dots, k_r)$. These Euler characteristics are then computed in Theorem 3.8 using the results of Harer and Zagier.

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2. EQUIVARIANT EULER CHARACTERISTICS

Let X be a complex quasi-projective variety with an action of a finite group G . Let us denote by $F(X, n)$ the configuration space of ordered n -tuples of distinct points on X . For each n , the action of the group G on X can be naturally extended to the action of G on $F(X, n)$, commuting with the natural action of S_n .

In the computations below we will use the additivity and multiplicativity of the Euler characteristic, as well as the Fubini formula for the integration with respect to the Euler characteristic ([15, 19], see also [16]).

Lemma 2.1. *The following equation holds: $\sum_{n=0}^{\infty} \frac{t^n}{n!} \chi(F(X, n)) = (1+t)^{\chi(X)}$.*

Proof. The map $\pi_n : F(X, n) \rightarrow F(X, n-1)$, which forgets the last point in the n -tuple, has fibers isomorphic to X without $n-1$ points. Therefore $\chi(F(X, n)) = (\chi(X) - n + 1) \cdot \chi(F(X, n-1))$, and $\chi(F(X, n)) = \chi(X) \cdot (\chi(X) - 1) \cdot \dots \cdot (\chi(X) - n + 1)$. \square

Let p_k denote the k th power sum and let V_λ denote the irreducible representation of S_n labelled by the Young diagram λ . We define the S_n -equivariant Euler characteristic of $F(X, n)/G$ by the equation

$$\chi^{S_n}(F(X, n)/G) = \sum_{i, \lambda} (-1)^i a_{i, \lambda} s_\lambda,$$

where $H^i(F(X, n)/G) = \bigoplus_\lambda a_{i, \lambda} V_\lambda$ and s_λ is the Schur polynomial.

Lemma 2.2. *The following equation holds:*

$$\chi^{S_n}(F(X, n)/G) = \frac{1}{n!} \sum_{\sigma \in S_n} p_1^{k_1(\sigma)} \cdot \dots \cdot p_n^{k_n(\sigma)} \cdot \chi([F(X, n)/G]^\sigma),$$

where $k_i(\sigma)$ is the number of cycles of length i in a permutation σ .

Proof. It is well known that for every i

$$\sum_\lambda a_{i, \lambda} s_\lambda = \frac{1}{n!} \sum_{\sigma \in S_n} p_1^{k_1(\sigma)} \cdot \dots \cdot p_n^{k_n(\sigma)} \cdot \text{Tr}(\sigma)|_{H^i(F(X, n)/G)},$$

hence

$$\chi^{S_n}(F(X, n)/G) = \frac{1}{n!} \sum_i (-1)^i \sum_{\sigma \in S_n} p_1^{k_1(\sigma)} \cdot \dots \cdot p_n^{k_n(\sigma)} \cdot \text{Tr}(\sigma)|_{H^i(F(X, n)/G)}$$

Now the statement follows from the Lefschetz fixed point theorem. \square

Lemma 2.3. *Let $\sigma \in S_n$. Then*

$$\chi([F(X, n)/G]^\sigma) = \frac{1}{|G|} \sum_{g \in G} \chi\left(F(X, n)^{g^{-1}\sigma}\right).$$

Proof. For a point $\mathbf{y} \in F(X, n)$ whose projection on $F(X, n)/G$ is σ -invariant there exists an element $g \in G$ such that $\sigma\mathbf{y} = g\mathbf{y}$. Consider the set of pairs

$$S = \{(g, \mathbf{y}) | g \in G, \mathbf{y} \in F(X, n) | \sigma\mathbf{y} = g\mathbf{y}\}$$

and its two-step projection $S \rightarrow F(X, n) \rightarrow F(X, n)/G$. The fiber of the first projection over a point \mathbf{y} is isomorphic to G -stabiliser of \mathbf{y} or empty, the fiber of the second projection containing \mathbf{y} is exactly the orbit of \mathbf{y} . Therefore the cardinality of every fiber of the composition is equal to $|G|$. \square

Definition 2.4. For any $g \in G$ we denote by $X_k(g)$ the subset of X consisting of points with g -orbits of length k . For example, $X_1(g)$ is a set of g -fixed points. Let $\tilde{X}_k(g) = X_k(g)/(g)$, where (g) is a cyclic subgroup in G generated by g .

The following theorem was deduced in [10] from the results of Getzler [6, 7], here we would like to present a more geometric and straightforward proof.

Theorem 2.5. *The generating function for the S_n -equivariant Euler characteristics of the quotients $F(X, n)/G$ is given by the following equation:*

$$(1) \quad \sum_{n=0}^{\infty} t^n \chi^{S_n}(F(X, n)/G) = \frac{1}{|G|} \sum_{g \in G} \prod_{k=1}^{\infty} (1 + p_k t^k)^{\frac{\chi(X_k(g))}{k}}.$$

Proof. Since all points in $X_k(g)$ have g -orbit of length k , we have $\chi(\tilde{X}_k(g)) = \chi(X_k(g))/k$. From Lemma 2.1 one gets:

$$(1 + p_j t^j)^{\chi(\tilde{X}_j(g))} = \sum_{k_j=0}^{\infty} \frac{p_j^{k_j} t^{j k_j}}{(k_j)!} \chi\left(F\left(\tilde{X}_j(g), k_j\right)\right),$$

Therefore the coefficient at t^n in the right hand side of (1) equals to:

$$\frac{1}{|G|} \sum_{g \in G} \sum_{\sum j k_j = n} \prod_j \frac{p_j^{k_j}}{k_j!} \chi\left(F\left(\tilde{X}_j(g), k_j\right)\right).$$

On the other hand, by Lemma 2.2 and Lemma 2.3, the left hand side of (1) can be rewritten as following:

$$\frac{1}{|G|} \sum_{g \in G} \frac{1}{n!} \sum_{\sigma \in S_n} p_1^{k_1(\sigma)} \cdot \dots \cdot p_n^{k_n(\sigma)} \cdot \chi([F(X, n)]^{g^{-1}\sigma}).$$

If for a tuple $\mathbf{y} \in F(X, n)$ we have $\sigma(\mathbf{y}) = g(\mathbf{y})$, the action of (g) at this tuple has $k_j(\sigma)$ cycles of length j . Every cycle of length j corresponds to a point in $\tilde{X}_j(g)$, hence for every g we can define a map

$$\alpha_g : \sqcup_{\sigma \in S_n} [F(X, n)]^{g^{-1}\sigma} \rightarrow \prod_j F(\tilde{X}_j(g), k_j)/S_{k_j}.$$

Given a g -invariant n -tuple of distinct points in X , there are $n!$ ways to label them and make an ordered tuple \mathbf{y} . Every such ordering defines a unique permutation σ such that $\sigma(\mathbf{y}) = g(\mathbf{y})$, therefore all fibers of α_g have cardinality $n!$ and

$$\frac{1}{n!} \sum_{\sigma \in S_n} \chi([F(X, n)]^{g^{-1}\sigma}) = \prod_j \chi\left(F(\tilde{X}_j(g), k_j)/S_{k_j}\right) = \prod_j \frac{\chi\left(F(\tilde{X}_j(g), k_j)\right)}{k_j!}.$$

□

3. MODULI SPACES OF CURVES

Let us apply Theorem 2.5 to the study of moduli spaces of curves. Let \mathcal{M}_g denote the moduli space of genus g algebraic curves and let $\mathcal{M}_{g,n}$ denote the moduli space of genus g algebraic curves with n parked points (we will always assume $g \geq 2$). Let $\mathcal{M}_g(k_1, \dots, k_r)$ be the moduli space of pairs (C, τ) where C is a genus g curve and τ is an automorphism of C such that $\chi(C_i(\tau)) = ik_i$ for all i . Since $g \geq 2$, every automorphism of C has finite order, hence one can choose r such that $k_r \neq 0$ and $k_i = 0$ for $i > r$.

There is a natural forgetful map $\pi_{g,\underline{k}} : \mathcal{M}_g(k_1, \dots, k_r) \rightarrow \mathcal{M}_g$ sending (C, τ) to C . For a curve C we define $\text{Aut}_{\underline{k}}(C) = \pi_{g,\underline{k}}^{-1}(C) \subset \text{Aut}(C)$.

Proposition 3.1. Suppose that $\mathcal{M}_g(k_1, \dots, k_r)$ is not empty. Then $k_r < 0$, $k_i = 0$ for $i \nmid r$ and $k_i \geq 0$ for $i \mid r$, $i < r$. Moreover, we have the following bounds on r and k_i :

$$r \leq 4g + 2, \quad |k_r| \leq 2g, \quad \sum_{i=1}^{r-1} k_i \leq 2g + 2.$$

Proof. Let τ be an automorphism of a genus g curve C such that $\chi(C_i(\tau)) = ik_i$ for all i . Note that $C_i(\tau)$ are finite sets for $i < r$ and

$$(2) \quad \chi(C) = 2 - 2g = \sum_{i=1}^{r-1} ik_i - r|k_r|$$

The quotient $C_1 = C/\tau$ is a smooth curve of some genus h , and the Riemann-Hurwitz formula yields its Euler characteristic:

$$(3) \quad \chi(C_1) = 2 - 2h = \sum_{i=1}^{r-1} k_i - |k_r|.$$

The projection of C to C_1 is a ramified covering of order r with $s = \sum_{i=1}^{r-1} k_j$ ramification points. The automorphism τ has order r , so $i \mid r$, if $k_i \neq 0$. By a theorem of Wiman ([20], see also [14]), the maximal order for an automorphism of a genus g curve equals $4g + 2$, hence $r \leq 4g + 2$.

Since proper divisors of r cannot exceed $r/2$, equation (3) implies:

$$\sum_{i=1}^{r-1} ik_i \leq \frac{r}{2} \sum_{i=1}^{r-1} k_i = \frac{r}{2}(2 - 2h + |k_r|),$$

hence by (2):

$$(4) \quad 2g - 2 = r|k_r| - \sum_{i=1}^{r-1} ik_i \geq \frac{r}{2}(2h + |k_r| - 2).$$

Therefore $|k_r| - 2 \leq 2g - 2$ and $|k_r| \leq 2g$. Finally, $\sum_{i=1}^{r-1} k_i = |k_r| + 2 - 2h \leq 2g + 2$. □

Remark 3.2. The bounds on r and on k_i are sharp. Indeed, consider a hyperelliptic curve P covering \mathbb{CP}^1 with ramifications at the vertices of a regular $(2g + 1)$ -gon and at its center. The covering can be chosen such that the automorphism of P induced by the rotation of this polygon acts nontrivially in the fibers and hence has order $r = 2(2g + 1) = 4g + 2$.

On the other hand, consider a hyperelliptic curve C with involution τ . We have

$$\chi(C_1(\tau)) = 2g + 2, \quad \chi(C_2(\tau)) = 2 - 2g - (2g + 2) = -4g,$$

hence a pair (C, τ) belongs to the moduli space $\mathcal{M}_g(2g + 2, -2g)$.

Theorem 3.3. *The following equation holds:*

$$(5) \quad \sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{\underline{k}} \chi^{orb}(\mathcal{M}_g(k_1, \dots, k_r)) \cdot \prod_{j=1}^r (1 + p_j t^j)^{k_j}.$$

Proof. Consider the forgetful map $\pi_{g,n} : \mathcal{M}_{g,n} \rightarrow \mathcal{M}_g$. Its fiber over a point representing a curve C is isomorphic to $F(C, n)/\text{Aut}(C)$, hence one can apply Theorem 2.5 to compute its equivariant Euler characteristic:

$$\begin{aligned} \sum_{n=0}^{\infty} t^n \chi^{S_n}(\pi_{g,n}^{-1}(C)) &= \sum_{n=0}^{\infty} t^n \chi^{S_n}(F(C, n)/\text{Aut}(C)) = \\ \frac{1}{|\text{Aut}(C)|} \sum_{\tau \in \text{Aut}(C)} \prod_i (1 + p_i t^i)^{\frac{\chi(C_i(\tau))}{i}} &= \sum_{\underline{k}} \frac{1}{|\text{Aut}(C)|} \sum_{\tau \in \text{Aut}_{\underline{k}}(C)} \prod_i (1 + p_i t^i)^{k_i}. \end{aligned}$$

Therefore:

$$\begin{aligned} \sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) &= \int_{\mathcal{M}_g} \sum_{n=0}^{\infty} t^n \chi^{S_n}(\pi_{g,n}^{-1}(C)) d\chi = \\ \sum_{\underline{k}} \prod_i (1 + p_i t^i)^{k_i} \int_{\mathcal{M}_g} \frac{|\text{Aut}_{\underline{k}}(C)|}{|\text{Aut}(C)|} d\chi. \end{aligned}$$

On the other hand,

$$\chi^{orb}(\mathcal{M}_g(k_1, \dots, k_r)) = \int_{\mathcal{M}_g} \frac{|\pi_{g,\underline{k}}^{-1}(C)|}{|\text{Aut}(C)|} d\chi = \int_{\mathcal{M}_g} \frac{|\text{Aut}_{\underline{k}}(C)|}{|\text{Aut}(C)|} d\chi \quad \square$$

Using the Proposition 3.1, we conclude that the sum in the right hand side of (5) is finite.

Corollary 3.4. The generating function $\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n})$ is a rational function in t . Furthermore, for any n ,

$$\chi^{S_n}(\mathcal{M}_{g,n}) \in \mathbb{Z}[p_1, \dots, p_{4g+2}].$$

The orbifold Euler characteristic of $\mathcal{M}_g(k_1, \dots, k_r)$ can be computed using the combinatorial results of Harer and Zagier [13]. We will denote the greatest common divisor of integers a and b by (a, b) . Let $\varphi(n)$ and $\mu(n)$ denote the Euler function and the Möbius function respectively. Define

$$c(k, l, d) := \mu\left(\frac{d}{(d, l)}\right) \frac{\varphi(k/l)}{\varphi(d/(d, l))},$$

Definition 3.5. Let $\lambda = (\lambda_1, \dots, \lambda_s)$ be a partition. We define a number

$$N(r; \lambda) = |\{(x_1, \dots, x_s) \in (\mathbb{Z}/r\mathbb{Z})^s : x_1 + \dots + x_s \equiv 0 \pmod{r}, (x_i, k) = \lambda_i\}|.$$

Lemma 3.6. ([13]) *The following equation holds:*

$$N(r; \lambda) = \frac{1}{r} \sum_{d|r} \varphi(d) \prod_{i=1}^s c(k, \lambda_i, d).$$

Theorem 3.7. ([13]) *The orbifold Euler characteristic of the moduli space $\mathcal{M}_{h,s}$ of genus h curves with s marked points is given by the formula:*

$$\chi^{orb}(\mathcal{M}_{h,s}) = (-1)^s \frac{(2h-1)B_{2h}}{(2h)!} (2h+s-3)!$$

where B_k denote Bernoulli numbers.

Theorem 3.8. *The generating function for the S_n -equivariant Euler characteristics of $\mathcal{M}_{g,n}$ has the form*

$$\sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{g,n}) = \sum_{\underline{k}} c_{k_1, \dots, k_r} \prod_{j=1}^r (1 + p_j t^j)^{k_j},$$

where p_j are power sums and the coefficients c_{k_1, \dots, k_r} are defined by the equation:

$$(6) \quad c_{k_1, \dots, k_r} = \chi^{orb}(\mathcal{M}_{h,s}) r^{2h} \prod_{p|\gamma} (1 - p^{-2h}) \cdot \frac{N(r; \lambda)}{r \prod_{i=1}^{r-1} k_i!}.$$

Here $h = \frac{1}{2}(1 - \sum_{j=1}^r k_j)$, $s = \sum_{j=1}^{r-1} k_j$, $\gamma = \text{GCD}(i : k_i > 0)$, $\lambda = (1^{k_1} 2^{k_2} \dots (r-1)^{k_{r-1}})$

Proof. By Theorem 3.3 one has $c_{k_1, \dots, k_r} = \chi^{orb}(\mathcal{M}_g(k_1, \dots, k_r))$. Consider the moduli space $\mathcal{M}_g(k_1, \dots, k_r)$ of pairs (C, τ) . As in Proposition 3.1, to such a pair one can associate a genus h curve $C_1 = C/\tau$. The projection from C to C_1 is ramified in s points subdivided into groups of size k_1, \dots, k_{r-1} . The orbifold Euler characteristic of the moduli space of genus h curves with such markings equals $\chi^{orb}(\mathcal{M}_{h,s}) / \prod_{i=1}^{r-1} k_i!$.

The number of pairs (C, τ) associated to a curve C_1 with fixed marked points was computed in [13, pages 478–479] and equals

$$\frac{1}{r} r^{2h} \prod_{p|\gamma} (1 - p^{-2h}) \cdot N(r; \lambda).$$

This completes the proof. □

The non-equivariant Euler characteristic of $\mathcal{M}_{g,n}$ has been computed in [3, Theorem 4.3]. It can be compared with Theorem 3.8 since

$$\chi(\mathcal{M}_{g,n}) = n! \cdot \chi^{S_n}(\mathcal{M}_{g,n})[p_1 = 1, p_k = 0 \text{ for } k > 1].$$

Example 3.9. The generating function for the S_n -equivariant Euler characteristics of the moduli spaces of genus 2 curves with marked points has a form [10]:

$$\begin{aligned} \sum_{n=0}^{\infty} t^n \chi^{S_n}(\mathcal{M}_{2,n}) = & -\frac{1}{240}(1 + p_1 t)^{-2} - \frac{1}{240}(1 + p_1 t)^6 (1 + p_2 t^2)^{-4} + \\ & + \frac{2}{5}(1 + p_1 t)^3 (1 + p_5 t^5)^{-1} + \frac{2}{5}(1 + p_1 t)(1 + p_2 t^2)(1 + p_5 t^5)(1 + p_{10} t^{10})^{-1} + \\ & + \frac{1}{6}(1 + p_1 t)^2 (1 + p_2 t^2)(1 + p_6 t^6)^{-1} - \frac{1}{12}(1 + p_1 t)^4 (1 + p_3 t^3)^{-2} - \\ & - \frac{1}{12}(1 + p_2 t^2)^2 (1 + p_3 t^3)^2 (1 + p_6 t^6)^{-2} + \frac{1}{12}(1 + p_1 t)^2 (1 + p_2 t^2)^{-2} + \\ & + \frac{1}{4}(1 + p_1 t)^2 (1 + p_4 t^4)(1 + p_8 t^8)^{-1} - \frac{1}{8}(1 + p_1 t)^2 (1 + p_2 t^2)^2 (1 + p_4 t^4)^{-2}. \end{aligned}$$

These coefficients can be matched with the ones defined in Theorem 3.8.

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